

The Dynamics of the
Outer Satellites
of Saturn

by

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For my dear friends Ruth, Ian and Richard whose kindness and encouragement during the last three years has been a source of inspiration.

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The numerical integration program 'TITAN' described in Chapter 5 was written by Dr. A.T. Sinclair. The description of the program given in section 5.2 is derived from Sinclair and Taylor (1985) and from Dr Sinclair's own notes. The version of the program used in this work is essentially the original version provided by Dr Sinclair, with several minor alterations to enable it to run correctly on the IBM VM/370 system of the University of Liverpool Computer Laboratory.

The observations of the satellites of Saturn used in this work were collected by Dr. D.B. Taylor and punched onto 80-column cards by Mrs. D.E. Oliver at the Royal Greenwich Observatory. As supplied by Dr Taylor, the data files were direct transcripts of the original references, the data being tabulated in several formats according to the format of the source reference. These data files were the raw material for the preparation whose theory is given in Chapter 4. In practise, the preparation involved the writing of several programs to re-format the data and to add auxiliary quantities. The final result is a set of files in a single format which incorporate the original data files plus all auxiliary quantities required for the analysis of the data. This work was carried out at the Royal Greenwich Observatory during the summer of 1985.

The first two figures in Chapter 2 are taken directly from Sinclair (1974) with the permission of Dr Sinclair.

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To [celestial mechanics] I was especially attracted because its preparation seemed to me to embody the highest intellectual power to which man had ever attained. The matter used to present itself to my mind somewhat in this way : Supply any man with the fundamental data of astronomy, the times at which stars and planets cross the meridian of a place, and other matters of this kind. He is informed that each of these bodies whose observations he is to use is attracted by all the others with a force which varies as the inverse square of their distances apart. From these data he is to weigh the bodies, predict their motion in all future time, compute their orbits, determine what changes of form and position these orbits will undergo through thousands of ages, and make maps showing exactly over what cities and towns on the surface of the earth an eclipse of the sun will pass fifty years hence, or over what regions it did pass thousands of years ago. A more hopeless problem than this could not be presented to the ordinary human intellect. There are tens of thousands of men who could be successful in all of the ordinary walks of life, hundreds who could wield empires, thousands who could gain wealth, for one who could take up this astronomical problem with any hope of success. The men who have done it are therefore in intellect the select few of the human race, -- an aristocracy ranking above all others in the scale of being. The astronomical ephemeris is the last practical outcome of their productive genius.

Simon Newcomb (1903) 'Reminiscences of an Astronomer' pp.63-4

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